Content

Course Code	Course Name	Semester	Theory	Practice	Lab	Credit	ECTS
MATH 504		1	3	0	0	3	7

Prerequisites	
Admission Requirements	

Language of Instruction	English
Course Type	Compulsory
Course Level	Masters Degree
Objective	Understanding the fundamental algebraic concepts and being able to use them within professional mathematics.
Content	The course has two parts. The first builds the general vocabulary of algebra: structures defined by their laws, quotients through universal properties, what passes from A to A[X], group actions, modules, tensor products. The second treats field extensions and Galois theory: algebraic closure, separability, the Galois correspondence, finite fields and cyclotomy, solvability by radicals. One result ties the parts together: the Hilbert basis theorem, proved for rings, is what the Nullstellensatz needs. Exercises belong to the exposition and are used by later results.
References	Serge Lang, Algebra; gitlab.com/onayg/mat504

Theory Topics

Week	Weekly Contents
1	Algebraic structures; quotients; congruences.
2	Noetherian rings and the Hilbert basis theorem; factorial rings and Gauss's theorem.
3	Group actions; orbit-stabiliser; class equation; Cauchy; Burnside.
4	Symmetric group; cycles; signature; simplicity of A_n for n ≥ 5.
5	Sylow's theorems; groups of small order.
6	Modules; Z-modules; K[T]-modules; free modules.
7	Torsion K[T]-modules; Jordan and rational forms; Cayley-Hamilton.
8	Tensor products; universal property; base change.
9	Field extensions; minimal polynomials; Zariski's lemma; Nullstellensatz.
10	Splitting fields; algebraic closure; extension of embeddings.
11	Separability; reduced base change; perfect fields; characteristic p.
12	Normal extensions; primitive element; Galois correspondence.
13	Finite fields; cyclotomic polynomials; Galois group (Z/nZ)^×.
14	Solvable groups; radical extensions; insolvability of the quintic.