Content

Course Code	Course Name	Semester	Theory	Practice	Lab	Credit	ECTS
MAT364	Introduction to Number Theory	6	3	0	0	3	5

Prerequisites	
Admission Requirements	
Language of Instruction	English
Course Type	Elective
Course Level	Bachelor Degree
Objective	In this course, the ultimate goal is to prove the quadratic reciprocity law using characters and Gauss sums.
Content	The course covers the fundamental concepts and key tools of the field, linking theory to applications through guided examples and progressively challenging exercises.
References	William Stein, Elementary Number Theory: Primes, Congruences, and Secrets, https://wstein.org/ent/ent.pdf Kenneth Ireland & Michael Rosen, A Classical Introduction to Modern Number Theory Ivan Niven, Herbert Zuckerman, Hugh Montgomery, An Introduction to the Theory of Numbers

Theory Topics

Week	Weekly Contents
1	Congruences; arithmetic mod n; guiding examples and conjectures.
2	Euclid's algorithm and B\'ezout's identity. GCD; modular inverses; linear congruences.
3	Prime numbers and unique factorization. Basic lemmas; applications to congruences.
4	The group $(\mathbb{Z}/n\mathbb{Z})^\times$. Euler's φ function; Euler's theorem; order of an element.
5	Quadratic residues: exploration. Squares modulo a prime; counting; first residue tables.
6	Legendre symbol and Euler's criterion. Definition; multiplicativity; fast computations.
7	Supplementary laws. $\left(\frac{-1}{p}\right)$ (mod 4) and $\left(\frac{2}{p}\right)$ (mod 8); guided proofs.
8	Multiplicative characters. Characters of $(\mathbb{Z}/p\mathbb{Z})^\times$; orthogonality; the quadratic character.
9	Additive characters and roots of unity. Exponential sums modulo p; basic identities.
10	Gauss sums I. Definition $\tau(\chi)$; twisting identities; absolute value and examples.
11	Gauss sums II. Evaluation of the quadratic Gauss sum; determining the sign via $p \bmod 4$.
12	Quadratic reciprocity. Proof via Gauss sums and characters; synthesis of the steps.
13	Jacobi symbol and effective computation. Generalization to composite denominators; caveats and examples.
14	Applications & projects. Deciding solvability of $x^2\equiv a \pmod p$; mini-projects and final portfolios.